

## Fall 2021 MAT 206.5 Final Review

**ONLY SCIENTIFIC CALCULATOR PERMITTED FOR THE FINAL EXAM**

1. Given  $\theta = \frac{7\pi}{6}$ ,
  - a. Convert  $\theta$  to degrees. [Review](#)
  - b. Draw  $\theta$  in the coordinate plane. [Review](#)
  - c. Name two angles, one positive and one negative, that are coterminal to  $\theta$ . [Review](#)
  - d. Determine the reference angle  $\hat{\theta}$ . [Review](#)
  - e. Find the exact value of  $\cos \theta$ . [Review](#)
  - f. Find the exact value of  $\cot \theta$ . [Review](#)
2. If  $\cos \theta = -\frac{\sqrt{2}}{3}$ , sketch  $\theta$  in the coordinate plane, and find  $\tan \theta$  and  $\csc \theta$ , where  $\theta$  terminates in the second quadrant.

Review this topic [here](#).

3. Sketch a graph of the given angle and evaluate the value of a trigonometric function of the angle without using a calculator.

a)  $\sin\left(-\frac{9\pi}{4}\right)$

b)  $\cot(-150^\circ) =$

Review this [topic](#).

4. Given  $f(x) = x^2 - 4$ ,  $g(x) = \sqrt{3-x}$ , Find the following:

a) The domain of  $f(x)$ . Write the answer in interval notation.

b) The domain of  $g(x)$ . Write the answer in interval notation.

c)  $(f \circ g)(x)$  and simplify.

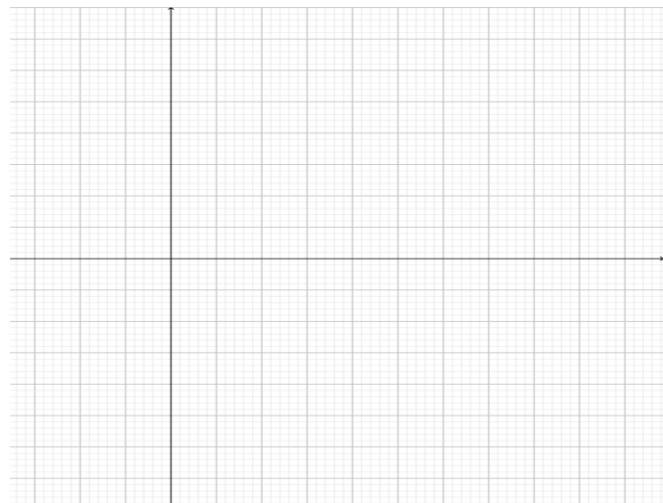
d) The domain of  $(f \circ g)(x)$ . Write the answer in interval notation.

Review this [topic](#).

5. Given  $f(x) = 4 \cos\left(3x - \frac{\pi}{2}\right) - 1$ ;

- a. What is the period of  $f(x)$ ?
- b. What is the amplitude of  $f(x)$ ?
- c. What is the phase shift of  $f(x)$ ?
- d. What is the equation of midline of  $f(x)$ ?
- e. What are 5 key points for one period of  $f(x)$ ?
- f. Sketch a graph of  $f(x)$  for two full periods.

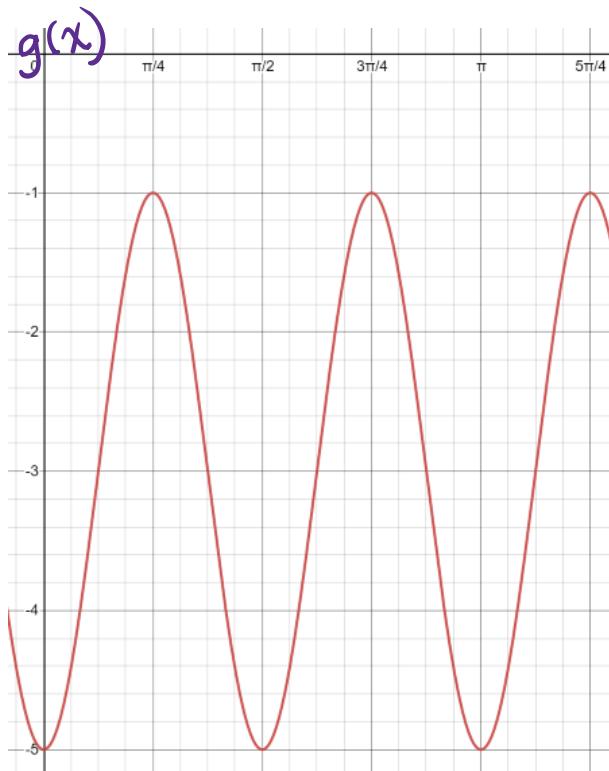
Review graphing sine/cosine functions [here](#).



6. The graph below  $g(x)$  is a transformation of the graph of  $f(x) = \sin(x)$ . There are no reflections in this graph from  $f(x)$ .

- a. What is the period of  $g(x)$ ?
- b. What is the amplitude of  $g(x)$ ?
- c. What is the phase shift of  $g(x)$ ?
- d. What is the equation of midline of  $g(x)$ ?
- e. What are 5 key points for one period of  $g(x)$ ?
- f. What is the equation of  $g(x)$ ?

Review graphing sine/cosine functions [here](#).

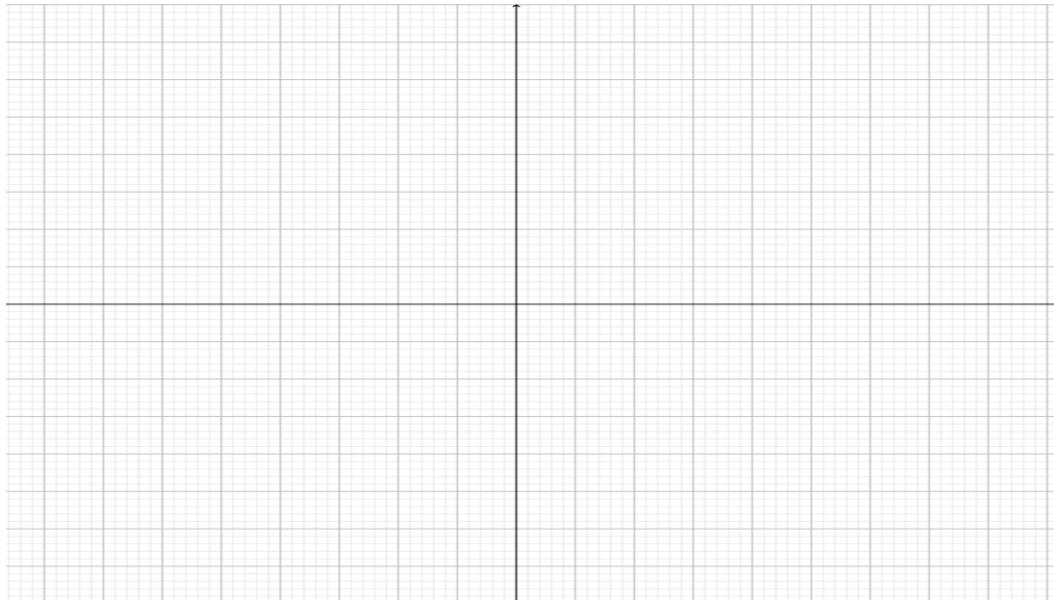


7. Using  $f(x) = 2x^2 - 3x$ , find  $\frac{f(-3+h)-f(-3)}{h}$ .

Review difference quotients [here](#).

8. Find the domain of  $f(x) = \frac{x^2 - 25}{x^2 + 3x - 40}$ , and identify any discontinuous point(s) (holes), vertical and/or horizontal asymptotes. Sketch a graph of  $f(x)$ .

Review graphing rational functions [here](#).



9. Write the partial fraction decomposition of the rational expression. Check your result algebraically.

a)  $\frac{1}{x^2+x}$

b)  $\frac{3}{x^2-3x}$

c)  $\frac{5}{x^2+x-6}$

d)  $\frac{x+1}{x^2-x-6}$

e)  $\frac{2x-3}{(x-1)^2}$

Review partial fractions [here](#).

10. Use synthetic division to find all real solutions of  $x^3 - 19x - 30 = 0$ .

Review synthetic division [here](#).

11. Find all the zeros of  $f(x) = 3x^3 - 2x^2 + 48x - 32$  given that  $\frac{2}{3}$  is a zero of  $f(x)$ .

Review finding zeros of a polynomial function [here](#).

12. Write as a single logarithm:

a)  $3\log_2 a + 2\log_2 b - \frac{1}{4}\log_2 c$

b)  $\ln x + 3\ln y$

c)  $2\log_3 a + \log_3(a - 3) - \log_3(a^2 - 9)$

Review condensing logarithmic expressions [here](#).

13. Find the exact value of each expression without using a calculator by using properties of logarithms (show your work!).

a)  $\log_4 \sqrt[5]{4}$

b)  $\ln e^{-10} + \ln e^2$

c)  $\log_4 32$

Review properties of logarithms [here](#).

14. Expand, using the properties of logarithms. Simplify if possible.

a)  $\log_5 \frac{4xy}{t}$

b)  $\log_2 \frac{x^3}{8\sqrt{y}}$

c)  $\ln \left( \frac{a^2 e^6}{b^{10}} \right)$

Review expanding logarithmic expressions [here](#).

15. Solve each equation. Round the answer to the nearest thousandth, if needed.

a)  $3(5^{2x+1}) = 75$     [Review](#)

b)  $14 + 2^{3a-5} = 197$     [Review](#)

c)  $\log_2 2x + \log_2(x + \frac{3}{2}) = 1$     [Review](#)

d)  $e^{2x} - 7e^x = 0$     [Review](#) (Hint: factor first!!)

e)  $\log_9 x + \log_9(2x - 3) = \log_9(x^2 + 4)$     [Review](#)

16. Evaluate the expression without using a calculator:

a)  $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

b)  $\arcsin 0$

c)  $\arccos\left(\cos\frac{\pi}{12}\right)$

d)  $\csc\left(\arccos\left(-\frac{3}{4}\right)\right)$

Review inverse trigonometric functions [here](#) (for c) and [here](#) (for a, b, and d).

17. Verify the identities:

a)  $\csc \alpha \cdot \tan \alpha = \frac{1}{\cos \alpha}$

b)  $\frac{1}{\cos \beta} = \cos \beta + \cos \beta \cdot \tan^2 \beta$

c)  $\frac{\sec^2 \theta}{1+\cot^2 \theta} = \tan^2 \theta$

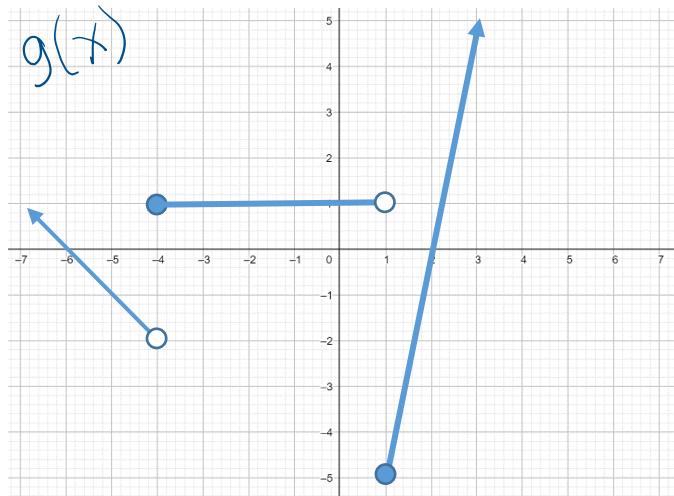
d)  $\frac{\cot^2 \theta}{\csc \theta + 1} = \frac{1-\sin \theta}{\sin \theta}$

Review verifying trigonometric identities [here](#).

18. Given  $5x - 2y = 11$ , write the (a) equation of the line that is parallel to it and passes through the point  $(-10, 3)$ , and (b) equation of the line that is perpendicular to it and passes through the point  $(-10, 3)$ . Write both equations in slope-intercept form.

Review writing equations of parallel and perpendicular lines [here](#). Just [parallel lines](#). Just [perpendicular lines](#).

19. Use the graph of  $g(x)$  to answer each question.



(a) Find the domain and range of  $g(x)$ . Write the answers in interval notation. [Review](#)

(b) Evaluate  $g(-3)$  and  $g(1)$ . [Review](#)

(c) On which interval(s) is the graph increasing, decreasing, or constant? [Review](#)

(d) What are the  $x$ - and  $y$ -intercepts of the graph? [Review](#)

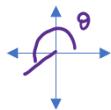
20. Given  $f(x) = x^2 - 3x - 4$  and  $g(x) = 2x^2 - 8x$ , find the following.

- $(f + g)(x)$
- $(f - g)(x)$
- $\frac{f}{g}(x)$ , and the domain

Review parts a and b [here](#). Review part c [here](#).

## Fall 2021 MAT 206.5 Final Review – Answer Key

1a.)  $210^\circ$



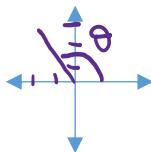
1c.) many answers, two are  $-\frac{5\pi}{6}$  and  $\frac{19\pi}{6}$   
 $(-150^\circ \text{ and } 570^\circ)$

1d.)  $\hat{\theta} = \frac{\pi}{6}$  (or  $30^\circ$ )

1e.)  $-\frac{\sqrt{3}}{2}$

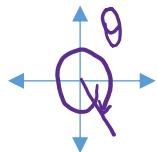
1f.)  $\sqrt{3}$

2.)

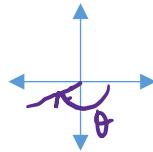


$\tan \theta = -\frac{\sqrt{14}}{2}$  and  $\csc \theta = \frac{3\sqrt{7}}{7}$

3a.)  $-\frac{\sqrt{2}}{2}$



3b.)  $\sqrt{3}$



4a.)  $(-\infty, \infty)$

4b.)  $(-\infty, 3]$

4c.)  $-x - 1$

4d.)  $(-\infty, 3]$

5a.)  $\frac{2\pi}{3}$

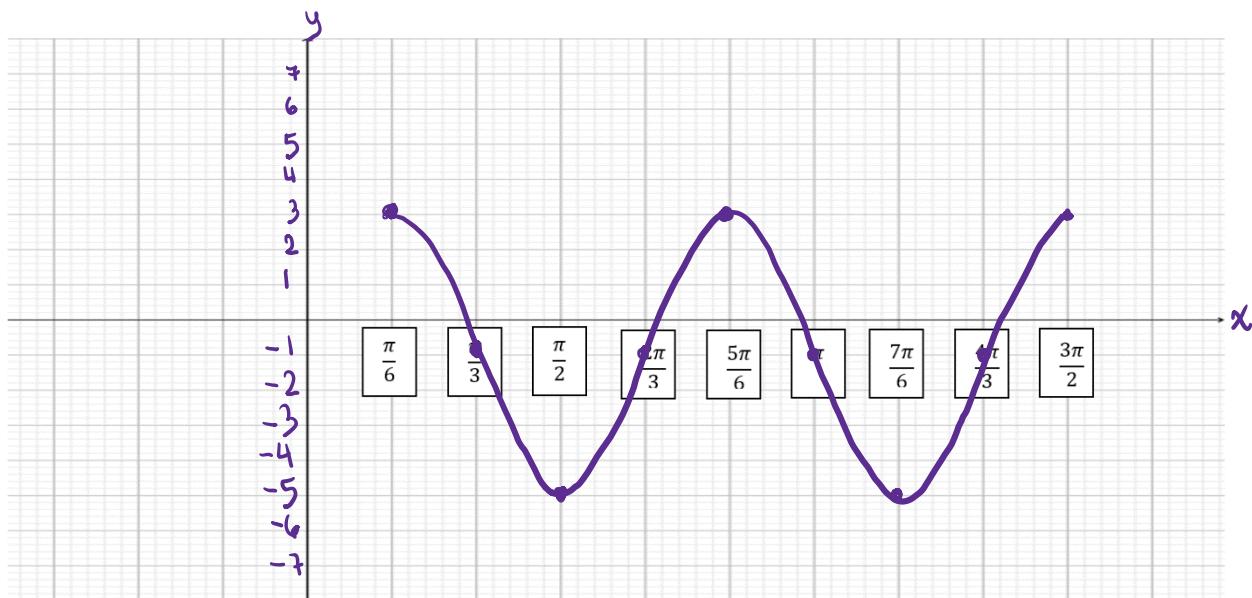
5b.)  $A = 4$

5c.) Phase shift =  $\frac{\pi}{6}$

5d.)  $y = -1$

5e.)

$x$	$\frac{\pi}{6}$	$\frac{\pi}{3}$ (or $\frac{2\pi}{6}$ )	$\frac{\pi}{2}$ (or $\frac{3\pi}{6}$ )	$\frac{2\pi}{3}$ (or $\frac{4\pi}{6}$ )	$\frac{5\pi}{3}$
$f(x)$	3	-1	-5	-1	3



$$6a.) \frac{\pi}{2}$$

$$6b.) 2$$

$$6c.) \frac{\pi}{8}$$

$$6d.) y = -3$$

6e.)

$x$	$\frac{\pi}{8}$	$\frac{\pi}{4}$ or $\frac{2\pi}{8}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$ or $\frac{4\pi}{8}$	$\frac{5\pi}{8}$
$g(x)$	-3	-1	-3	-5	-3

$$6f.) g(x) = 2 \sin\left(4x - \frac{\pi}{2}\right) - 3$$

$$7.) 2h - 15$$

8.) Domain:  $(-\infty, -8) \cup (-8, 5) \cup (5, \infty)$

Hole located at  $\left(5, \frac{10}{13}\right)$

Vertical asymptote at  $x = -8$

Horizontal asymptote at  $y = 1$



$$9a.) \frac{1}{x} - \frac{1}{x+1}$$

$$9b.) \frac{1}{x-3} - \frac{1}{x}$$

$$9c.) \frac{1}{x-2} - \frac{1}{x+3}$$

$$9d.) \frac{1}{5(x+2)} + \frac{4}{5(x-3)}$$

$$9e.) \frac{2}{x-1} - \frac{1}{(x-1)^2}$$

$$10.) x = \{-2, -3, 5\}$$

$$11.) x = \left\{\frac{2}{3}, 4i, -4i\right\}$$

$$12a.) \log_2 \frac{a^3 b^2}{\sqrt[4]{c}}$$

$$12b.) \ln(xy^3)$$

$$12c.) \log_3 \frac{a^2}{a+3}$$

$$13a.) \frac{1}{5}$$

$$13b.) -8$$

$$13c.) \frac{5}{2}$$

$$14a.) \log_5 4 + \log_5 x + \log_5 y - \log_5 t$$

$$14b.) 3 \log_2 x - 3 - \frac{1}{2} \log_2 y$$

$$14c.) 2 \ln a + 6 - 10 \ln b$$

$$15a.) x = \frac{1}{2}$$

$$15b.) x \approx 4.172$$

$$15c.) x = \frac{1}{2}$$

$$15d.) x \approx 1.946$$

$$15e.) x = 4$$

$$16a.) \frac{3\pi}{4}$$

$$16b.) 0$$

$$16c.) \frac{\pi}{12}$$

$$16d.) \frac{4\sqrt{7}}{7}$$

17a.) There are many correct answers! This is one correct answer.

$$\frac{1}{\cancel{\sin \alpha}} \cdot \frac{\cancel{\sin \alpha}}{\cos \alpha}$$

$$= \frac{1}{\cos \alpha}$$

17b.) There are many correct answers! This is one correct answer.

$$\begin{aligned} & \cos\beta \cdot (1 + \tan^2\beta) \\ &= \cos\beta \cdot \sec^2\beta \\ &= \cos\beta \cdot \frac{1}{\cos^2\beta} \\ &= \frac{1}{\cos^2\beta} \end{aligned}$$

17c.) There are many correct answers! This is one correct answer.

$$\begin{aligned} \Rightarrow \frac{\sec^2\theta}{\csc^2\theta} &= \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}} \\ &= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta \end{aligned}$$

17d.) There are many correct answers! This is one correct answer.

$$\begin{aligned} & \frac{\csc^2\theta - 1}{\csc\theta + 1} \\ \Rightarrow & \frac{(\csc\theta + 1)(\csc\theta - 1)}{\csc\theta + 1} \\ \Rightarrow & \csc\theta - 1 \\ &= \frac{1}{\sin\theta} - 1 \Rightarrow \frac{1}{\sin\theta} - \frac{\sin\theta}{\sin\theta} \\ &= \frac{1 - \sin\theta}{\sin\theta} \end{aligned}$$

18.) Equation of parallel line:  $y = \frac{5}{2}x + 28$

Equation of perpendicular line:  $y = -\frac{2}{5}x - 1$

19a.) Domain:  $(-\infty, \infty)$ ; Range:  $[-5, \infty)$

19b.)  $g(-3) = 1$ ;  $g(1) = -5$

19c.) Increasing:  $[1, \infty)$ ; Decreasing:  $(-\infty, -4)$ ; Constant:  $[-4, 1)$

19d.) x-intercepts:  $(-6, 0)$  and  $(2, 0)$ ; y-intercept:  $(0, 1)$

20a.)  $3x^2 - 11x - 4$

20b.)  $-x^2 + 5x - 4$

20c.)  $\frac{x+1}{2x}$  and the domain is  $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$